

THE BOUNDS ON LORENTZ AND CPT VIOLATING PARAMETERS IN THE HIGGS SECTOR

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In this talk, I discuss possible bounds on the Lorentz and CPT violating parameters in the Higgs sector of the so called minimal standard model extension. The main motivation to this study is coming from the fact that unlike the parameters in the fermion and gauge sector, there are no published bounds on the parameters in the Higgs sector. From the one-loop contributions to the photon propagator the bounds on the CPT-even asymmetric coefficients are obtained and the $c_{\mu\nu}$ coefficients in the fermion sector determine the bound on the CPT-even symmetric coefficients. The CPT-odd coefficient is bounded from the non-zero vacuum expectation value of the Z -boson.

1. Introduction

Lorentz and CPT symmetries are assumed to be exact in nature within the framework of the standard model and this fact is in very good agreement to high precision with present-day experimental findings. However, it is widely believed that the standard model is nothing but a low energy version of some more complete (fundamental) theory, presumably valid at the Planck scale of 10^{19} GeV, such as noncommutative field theory¹ or string theory.² It is then reasonable to search for some induced “new physics” effects at levels attainable by high precision experiments. The violation of Lorentz and CPT symmetries can be considered one of such effects.

There is an explicit example from string theory in which non locality of the string leads to modification of the Lorentz properties of the vacuum. Among mechanisms to describe Lorentz and CPT violation, the most elegant way is to consider these symmetries exact at the scale of the fundamental theory and spontaneously broken at low energies due to the existence of nonvanishing expectation value of some background tensor fields.

The 4-dimensional effective interactions between the background tensor

fields T and matter can be written as³

$$L' \supset \frac{\lambda}{M_{pl}^k} \langle T \rangle \cdot \bar{\psi} \Gamma (i\partial)^k \psi + H.c., \quad k \leq 2 \quad (1)$$

where all possible Lorentz indices are suppressed. For $k = 0, 1$, the first two factors of the right hand side of Eq. (1) represent most of the CPT-violating terms in the fermion sector. At this point it is better to explain the difference between the *observer* Lorentz invariance and the *particle* Lorentz invariance, which are essential for understanding the minimal standard model-extension (SME) that I will describe briefly in the next section. The former involves transformations under rotations and boosts of coordinate system but the latter involves boosts on particle or localized fields but not on the background fields. Therefore, while, in the right hand side of Eq. (1), $\langle T \rangle$ and $\bar{\psi} \Gamma i\partial \psi$ are both changing under *observer* Lorentz transformation such that their contraction stays invariant, *particle* Lorentz transformation leaves $\langle T \rangle$ term unaffected which leads to a (particle) Lorentz violating effect when it is contracted with the matter term. The following example from conventional electrodynamics³ can be given to give further clarification. Let us consider a charged particle entering a region perpendicular to a uniform background magnetic field. Its path is circular. Suppose without changing the observer frame, one gives an instantaneous particle boost to the charged particle without effecting its direction. Then it will still keep moving on a circular path but with a bigger or smaller radius depending on the direction of the given boost. This boost leaves the background magnetic field unaffected (here, the background magnetic field is analogous to the field T). Let us now consider another observer frame which is obtained from our original frame by making a Lorentz transformation of coordinates. In that frame, the particle no longer makes a circular motion but a spiral motion (drift motion) due to the existence of induced electric field in addition to the magnetic field. The background field is obviously not a pure magnetic field at all. The important point is that the background field is changing to preserve the observer invariance, i.e. $F_{\mu\nu} F^{\mu\nu}$ term is invariant. This means that any Lorentz indices in each term of Eq. (1) must be contracted.

The outline of the talk, which is based on work done with David L. Anderson and Marc Sher⁴, is as follows. In Sec. 2, I will very briefly describe the minimal standard model extension by especially emphasizing its fermion, photon and Higgs sector. The purpose of our study is to explore the bounds on the parameters appearing in the Higgs sector of the minimal

SME. So, in Sec. 3, I consider the bounds on the CPT-even antisymmetric and symmetric coefficients of the Higgs sector. A careful analysis of the coordinate and field redefinition issue will be done. The bounds on CPT-odd coefficient in the same sector are discussed in Sec. 4.

2. The minimal Standard Model-Extension

A framework for studying Lorentz and CPT violation has been constructed by Colladay and Kostelecký⁵, known as the minimal SME. It is a model based on the standard model but which relaxes the Lorentz and CPT invariance. The additional induced terms representing such violation are still invariant under $SU(3) \times SU(2) \times U(1)$ gauge group of the standard model. As explained earlier, they preserve the observer Lorentz invariance but not the particle Lorentz invariance. The parameters in the minimal SME are assumed to be constant over space-time and this is the reason why we call it “minimal”. An extension of the model by including gravity in the context of some non-Minkowski spacetimes has been recently discussed by Kostelecký⁶ and the parameters become spacetime dependent.

As an example, for simplicity, the QED sector of the minimal SME which involves the electron and photon sectors is given here.

$$L_f = \frac{1}{2} i\bar{\psi} \Gamma^\mu \overset{\leftrightarrow}{D}_\mu \psi - \bar{\psi} M \psi,$$

where Γ^μ and M denote

$$\begin{aligned} \Gamma^\mu &= \gamma^\mu + \Gamma_1^\mu, \\ \Gamma_1^\mu &\equiv c^{\nu\mu}\gamma_\nu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\mu + i f^\mu\gamma_5 + \frac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu}, \\ M &= m + M_1, \\ M_1 &\equiv a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}. \end{aligned}$$

Here all constants a, b, \dots, g and H represent expectation values of some background tensor fields and break the particle Lorentz invariance. The photon sector is given as

$$L_p = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu}, \quad (2)$$

where the Lorentz violation is represented by k_F and k_{AF} terms. The parameters have some properties. Let us quote some of them here. All terms in M_1 and k_{AF} have dimension of mass while all terms in Γ_1^μ and k_F are dimensionless. $(k_F)_{\kappa\lambda\mu\nu}$ is antisymmetric with respect to first two

and last two indices separately and it satisfies the double-trace condition, $(k_F)_{\mu\nu}^{\mu\nu} = 0$, to be sure that the photon field is normalized properly. Only b_μ, c_μ and $(k_F)_{\kappa\lambda\mu\nu}$ will be relevant to our discussion here and there are many experimental and theoretical talks about them in this meeting.

The Higgs sector is

$$\begin{aligned} L_{\text{Higgs}} &= (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{3!} (\Phi^\dagger \Phi)^2 + L'_{\text{Higgs}}^{\text{CPT-even}} + L'_{\text{Higgs}}^{\text{CPT-odd}}, \\ L'_{\text{Higgs}}^{\text{CPT-even}} &= \left[\frac{1}{2} (k_{\phi\phi}^S + ik_{\phi\phi}^A)_{\mu\nu} (D^\mu \Phi)^\dagger D^\nu \Phi + \text{H.c.} \right] - \frac{1}{2} k_{\phi B}^{\mu\nu} \Phi^\dagger \Phi B_{\mu\nu} \\ &\quad - \frac{1}{2} k_{\phi W}^{\mu\nu} \Phi^\dagger W_{\mu\nu} \Phi, \\ L'_{\text{Higgs}}^{\text{CPT-odd}} &= ik_\phi^\mu \Phi^\dagger D_\mu \Phi + \text{H.c.}, \end{aligned}$$

where $k_{\phi\phi}$ has real symmetric and imaginary antisymmetric parts, which are separated as above and $k_{\phi B}$ and $k_{\phi W}$ have only real symmetric parts. All are CPT preserving (CPT-even) but Lorentz violating and dimensionless. The only CPT-odd and mass dimension coefficient is k_ϕ .

3. The CPT-even coefficients

3.1. The CPT-even antisymmetric coefficients

Direct detection of these coefficients would necessitate producing large numbers of Higgs bosons, and the resulting bounds would be quite weak. However, there are extremely stringent bounds on Lorentz violation at low energies, and thus searching for the effects of these new interactions through loop effects will provide the strongest bounds. The most promising of these effects will be on the photon propagator.

In this section, we will consider the bounds on the CPT-even antisymmetric coefficients, $k_{\phi\phi}^A, k_{\phi B}$ and $k_{\phi W}$. We first look at the most general CPT-even photon propagator, and then relate the $k_{\phi\phi}^A$ coefficients to the Lorentz-violating terms in the photon propagator. Then, the experimental constraints on such terms lead directly to stringent bounds on the $k_{\phi\phi}^A$ coefficients. We then consider the $k_{\phi B}$ and $k_{\phi W}$ coefficients.

The equation of motion from the Lagrangian Eq. (2) is^a

$$M^{\alpha\delta} A_\delta = 0, \quad (3)$$

^aWe set k_{AF} -term to zero, since it is very tightly constrained from astrophysical observations.⁵

where

$$M^{\alpha\delta}(p) \equiv g^{\alpha\delta}p^2 - p^\alpha p^\delta - 2(k_F)^{\alpha\beta\gamma\delta}p_\beta p_\gamma. \quad (4)$$

The propagator is clearly gauge invariant (recall that k_F is antisymmetric under exchange of the first or last two indices). Note that while the $g^{\mu\nu}p^2 - p^\mu p^\nu$ structure is mandated by gauge invariance, the k_F term is separately gauge invariant and may differ order by order in perturbation theory. For simplicity, we look at the divergent parts of the one loop diagrams only. Consideration of higher orders and finite parts will give similar, although not necessarily identical, results. We can consider each of the possible terms independently by assuming that there is no high-precision cancellations. Let us start with $k_{\phi\phi}^A$.

The $k_{\phi\phi}^A$ -term leads to photon-Goldstone boson- W boson and photon-Goldstone boson-Goldstone boson type interactions which are absent in the conventional Standard Model. As we do in the Standard Model, it is possible to fix the gauge to simplify the calculations. The Standard Model gauge fixing removes the mixing between W^\mp boson and the charged Goldstone boson ϕ^\pm . A similar situation happens in the minimal SME if one modifies the gauge fixing functions by adding a $i(k_{\phi\phi}^A)_{\mu\nu}\partial^\mu A_i^\nu$ term to the $SU(2)$ functions and a similar $i(k_{\phi\phi}^A)_{\mu\nu}\partial^\mu B^\nu$ to the $U(1)$ function.⁴ However, such generalization also leads to an unwanted mixing between the gauge boson Z_μ and the derivative of the Higgs field, $\partial_\nu\phi_1$, which is contracted with $(k_{\phi\phi}^A)^{\mu\nu}$, as well as substantially complicating the photon propagator. An easier way is to use a mixed propagator of the form

Another feature of the $k_{\phi\phi}^A$ -term is the modification of the W -boson propagator. Up to the second order in $k_{\phi\phi}^A$, the propagator in the 't Hooft-Feynman gauge takes the form

$$i\Delta_{\nu\lambda}(\xi = 1) = i\Delta_{\nu\lambda}^{(0)} + m_W^2 \frac{(k_{\phi\phi}^A)_{\nu\lambda}}{(q^2 - m_W^2)^2} + im_W^4 \frac{(k_{\phi\phi}^A)_{\nu\alpha}(k_{\phi\phi}^A)^\alpha_\lambda}{(q^2 - m_W^2)^3}. \quad (5)$$

The one-loop contributions to the photon vacuum polarization are given in Fig. 1. Here we only include diagrams with second order $k_{\phi\phi}^A$, since one can show that all diagrams with one $k_{\phi\phi}^A$ inclusion vanish.

There are two possible structures in second order, which are either $(k_{\phi\phi}^A)_{\mu\lambda}(k_{\phi\phi}^A)^\lambda_\nu$ or $(k_{\phi\phi}^A)_{\mu\lambda}(k_{\phi\phi}^A)_{\lambda'\nu}p^\lambda p^{\lambda'}$. Here p is the four momentum

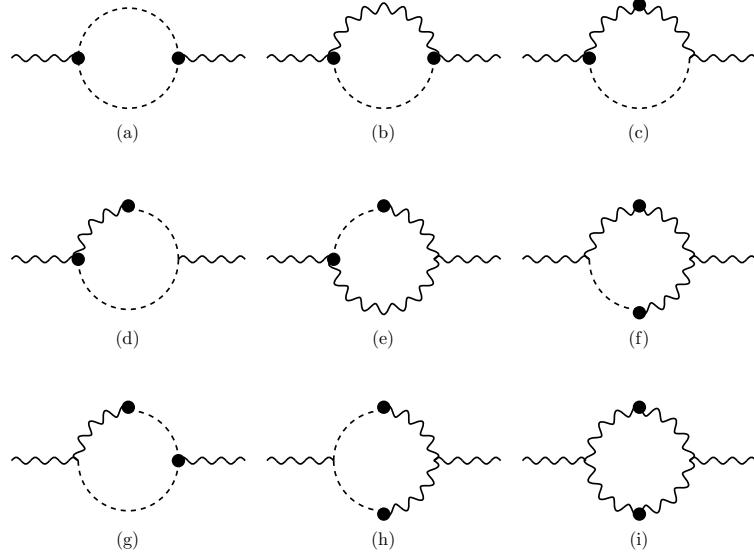


Figure 1. One-loop contributions to the photon vacuum polarization involving Lorentz-violating interactions to second order. These diagrams are for $k_{\phi\phi}^A$ case but similar diagrams exist for the other antisymmetric coefficients. Here the wavy (dashed) line circulating in the loop represents W boson (charged Goldstone boson). Each blob in vertices, W -propagator or W - ϕ mixed propagator represents a single Lorentz-violating coefficient insertion. The rest of the diagrams can be obtained by permutations of these 9 diagrams.

of the external photons. Again the first possibility is not gauge invariant and should vanish, thus contributions from the third term in Eq. (5) should vanish. We have verified this explicitly. The latter is gauge invariant and gives a non-zero contribution (if we contract with any of two external momenta of photons, p^μ or p^ν , it vanishes due to the antisymmetry property of $k_{\phi\phi}^A$). Calculating the one-loop diagrams, and comparing with Eq. (4), we find that the components of k_F can simply be expressed in terms of $k_{\phi\phi}^A$ as $(k_F)_{\mu\lambda\lambda'\nu} = \frac{1}{3}(k_{\phi\phi}^A)_{\mu\lambda}(k_{\phi\phi}^A)_{\lambda'\nu}$. We now turn to the experimental bounds on the k_F . Many speakers in this meeting have talked about the $\tilde{\kappa}_{e+}$ and by $\tilde{\kappa}_{o-}$ which are 3×3 matrices defined from the components of k_F and represent 10 of 19 elements of k_F .⁷ The strongest bound is coming from birefringence constraints⁷ and is given by 3×10^{-32} . I should note that for any single or possible combination of non-zero elements of $(k_{\phi\phi}^A)_{\mu\nu}$ it is impossible for both $\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$ to be null matrices, and thus the birefringence constraints apply.⁴ Therefore the upper bound of the $(k_{\phi\phi}^A)_{\mu\nu}$ coefficients can be obtained as 3×10^{-16} .

The discussion of $k_{\phi B}^{\mu\nu}$ and $k_{\phi W}^{\mu\nu}$ is very parallel to the $k_{\phi\phi}^A$ case. $k_{\phi B}^{\mu\nu}$ term does not induce a W -Goldstone mixing but leads to photon-Higgs scalar mixing instead. The $k_{\phi W}^{\mu\nu}$ term has very similar features to the $k_{\phi\phi}^A$ case except for the photon-Higgs-boson mixing. The $(k_F)_{\mu\lambda\lambda'\nu} = \frac{5}{12e^2} \cos^2 \theta_W (k_{\phi B})_{\mu\lambda} (k_{\phi B})_{\lambda'\nu}$ and $(k_F)_{\mu\lambda\lambda'\nu} = -\frac{5}{12e^2} \sin^2 \theta_W (k_{\phi W})_{\mu\lambda} (k_{\phi W})_{\lambda'\nu}$ equalities hold, which sets the bound as 0.9×10^{-16} and 1.7×10^{-16} , respectively. It is seen that the current bound on all three Lorentz violating coefficients is of the order of 10^{-16} and can easily be updated as the bound on k_F is updated.

3.2. Coordinate and field redefinitions and the symmetric coefficients

We consider bounds on the $k_{\phi\phi}^S$ coefficients. In this case, the strongest bounds come from relating, through field redefinitions, these coefficients to other Lorentz violating coefficients in the fermion sector, and then using previously determined bounds on those coefficients. Therefore, Coordinate and field redefinitions need to be discussed carefully.

Once any model is extended by relaxing some its symmetry properties, not all of the new parameters representing an apparent violation of these symmetries may be physical. That is, the model has some redundant parameters. Therefore, the extended model should be carefully analyzed to check for redundant parameters. This analysis may yield several Lagrangians which are equivalent to each other by some coordinate and field redefinitions and rescalings^{5,8,9,10}. The same situation applies to the minimal SME case. A simple observation from the fermion sector is that $\bar{\psi}\gamma^\mu D_\mu \psi - a_\mu \bar{\psi}\gamma^\mu \psi \rightarrow \bar{\psi}\gamma^\mu D_\mu \psi$ under $\psi \rightarrow \exp(-ia^\mu x_\mu)\psi$. Thus, a_μ is redundant unless gravity is included. A similar conclusion can be drawn for some components of $k_{\phi\phi}^S$ in the Higgs sector under certain circumstances. Consider a case^{7,10} with only two Lorentz-violating parameters $k_{\phi\phi}$ and k_F in the scalar and photon sectors, respectively. The Lagrangian is $L = [g_{\mu\nu} + (k_{\phi\phi})_{\mu\nu}] (D^\mu \Phi)^\dagger D^\nu \Phi - m^2 \Phi^\dagger \Phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\mu\lambda\lambda'\nu} F^{\mu\lambda} F^{\lambda'\nu}$, where $D_\mu = \partial_\mu + iqA_\mu$ and $k_{\phi\phi}$ is real and symmetric. Let us assume that only one component of $k_{\phi\phi}$, $(k_{\phi\phi})_{00} \equiv k^2 - 1$, is nonzero^{7,10} and that k_F is taken as zero. The transformations $t \rightarrow kt$, $\mathbf{x} \rightarrow \mathbf{x}$ and the field redefinitions $A_0 \rightarrow A_0$, $\mathbf{A} \rightarrow k\mathbf{A}$ with rescaling of the electric charge $q \rightarrow q/k$ move the Lorentz violation into the photon sector ($L_{\text{photon}} = (D_\mu \Phi)^\dagger D^\mu \Phi - m^2 \Phi^\dagger \Phi + \frac{1}{2} (E^2 - k^2 B^2)$, where $E(B)$ is the electric(magnetic) field). One further example is the following: Consider only $(k_{\phi\phi})_{11} = (k_{\phi\phi})_{22} = (k_{\phi\phi})_{33} = k^2 - 1$

nonzero and then it is still possible to get an equivalent Lagrangian as $L_{\text{photon}} = (D_\mu \Phi)^\dagger D^\mu \Phi - m^2 \Phi^\dagger \Phi + \frac{1}{2}(E^2 - B^2/k^2)$ under the transformations $t \rightarrow t$, $\mathbf{x} \rightarrow k\mathbf{x}$ and the redefinitions $A_0 \rightarrow kA_0$, $\mathbf{A} \rightarrow \mathbf{A}$ with the same charge rescaling $q \rightarrow q/k$. However, for the other components of $k_{\phi\phi}$, there are no such obvious transformations.

Another observation is from the electron sector of the extended QED⁸. The free electron Lagrangian with explicit Lorentz violation $L_f^0(\psi(x))$ transforms into $L_f^0(\chi(x)) + i c_{\mu\nu} \bar{\chi} \gamma^\mu \partial^\nu \chi = L_f^0(\chi(x'))$ under the transformation $\psi(x) = (1 + c_{\mu\nu} x^\mu \partial^\nu) \chi(x)$ (i.e., $x^\mu \rightarrow x'^\mu = x^\mu + c_\nu^\mu x^\nu$). Note that $c_{\mu\nu}$ is redundant unless fermion-photon interactions are included. Similarly the field redefinition of the Higgs doublet $\Phi(x) = \left(1 + \frac{1}{2}(k_{\phi\phi}^S)_{\mu\nu} x^\mu \partial^\nu\right) \varphi(x)$ eliminates the explicit Lorentz violation in the Higgs sector but the $(k_{\phi\phi}^S)$ -term reappears as a c -term in the photon sector. Thus, the redundancy of the parameters in the minimal SME is a matter of convention. Assuming a conventional fermion sector (and the photon sector in the case of including fermion-photon interactions) makes the $(k_{\phi\phi}^S)_{\mu\nu}$ physical. Otherwise, there is mixing among $k_{\phi\phi}^S$, $c_{\mu\nu}$, and nine unbounded k_F coefficients. In this study, we *only* concentrate on the Lorentz and CPT violation in the scalar sector of the SME, hence we assume that the theory has a conventional fermion sector, which means that bounds on $c_{\mu\nu}$ will lead to effective bounds on $k_{\phi\phi}^S$. The best current bounds on the components of $c_{\mu\nu}$ are summarized in Table 1 as direct bounds on the components of $(k_{\phi\phi}^S)_{\mu\nu}$. In general, we prefer using the measured cleaner bounds, if available, to some projected tighter bounds estimated from some planned experiments.

4. The CPT-odd coefficient

One interesting effect of the CPT-odd k_ϕ -term is the modification of the conventional electroweak $SU(2) \times U(1)$ symmetry breaking. Minimization of the static potential yields a nonzero expectation value for Z_μ boson field of the form $\langle Z_\mu \rangle_0 = \frac{\sin 2\theta_W}{q} \text{Re}(k_\phi)_\mu$.^b The nonzero expectation value for the Z will, when plugged into the conventional fermion-fermion- Z interaction, yield a $b_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi$ term.^c Then the relation $b_\mu = \frac{1}{4} \text{Re}(k_\phi)_\mu$ holds. The best bound on b_μ for its X and Y components comes from the neutron with the use of a two-species noble-gas maser¹⁹ and it is of the order of $b_{X,Y}^n \leq 10^{-32}$ GeV. Note that in order to get this bound there are some assumptions about

^bHere we have assumed all the other Lorentz-violating coefficients zero.

^cAlternatively, one can look at the one-loop effects on the photon propagator, however this will yield much weaker bounds.

the nuclear configurations, which make the bound uncertain accuracy to within one or two orders of magnitude. Details of the experiment and some new improvements can be found in the proceedings of both this and the previous meetings. The best bound on the Z component of b_μ comes from testing of cosmic spatial isotropy for polarized electrons²⁰ and it is of the order of $b_Z^e \leq 7.1 \times 10^{-28}$ GeV in the Sun-centered frame. The bound on the time component of b_μ is around $b_T^n \leq 10^{-27}$ GeV²¹. The complete list of all bounds on the Lorentz and CPT violating parameters of the Higgs sector is given in Table 1.

Parameters	Sources		b_μ (GeV)	Comments
	$\tilde{\kappa}_{e+}, \tilde{\kappa}_{o-}$	$c_{\mu\nu}$		
$(k_{\phi\phi}^A)_{\mu\nu}$	3×10^{-16}	-	-	-
$(k_{\phi B})_{\mu\nu}$	0.9×10^{-16}	-	-	-
$(k_{\phi W})_{\mu\nu}$	1.7×10^{-16}	-	-	-
$(k_{\phi\phi}^S)_{II}$	-	10^{-27}	-	a
$(k_{\phi\phi}^S)_{TT}$	-	4×10^{-13}	-	b
$(k_{\phi\phi}^S)_{TI}$	-	10^{-25}	-	c
$(k_{\phi\phi}^S)_{XZ}, (k_{\phi\phi}^S)_{YZ}$	-	10^{-25}	-	d
$(k_{\phi\phi}^S)_{XY}$	-	10^{-27}	-	d
$(k_\phi)_X, (k_\phi)_Y$	-	-	10^{-31}	e
$(k_\phi)_Z, (k_\phi)_T$	-	-	2.8×10^{-27}	f

Note:

- a) Obtained from $c_{\mu\nu}^{\text{neutron}}$ with the assumption that Lorentz violation is not isotropic.^{11,12,13} If it is isotropic, the bound on $(k_{\phi\phi}^S)_{TT}$ applies.¹⁴
- b) Obtained from the comparison of the anti-proton's frequency with the hydrogen ion's frequency.¹⁴
- c) Estimated value based on the sensitivity calculations of some planned space-experiments.^{12,15,16,17}
- d) Obtained from the neutron.^{18,11,12,13}
- e) From b_μ^{neutron} with the use of a two-species noble-gas maser. From b_μ^{electron} , a weaker but cleaner bound of 1.2×10^{-25} can be obtained.
- f) This bound is from the spatial isotropy test of polarized electrons.

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$(k_{\phi\phi}^S)_{II}$	-	10^{-27}	-	^a
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$(k_{\phi\phi}^S)_{XZ}, (k_{\phi\phi}^S)_{YZ}$	-	10^{-25}	-	^d
$(k_{\phi\phi}^S)_{XY}$	-	10^{-27}	-	^d
$(k_\phi)_X, (k_\phi)_Y$	-	-	10^{-31}	^e
$(k_\phi)_Z, (k_\phi)_T$	-	-	2.8×10^{-27}	^f

^aObtained from $c_{\mu\nu}^{\text{neutron}}$ with the assumption that Lorentz violation is not isotropic. If it is isotropic, the bound on $(k_{\phi\phi}^S)_{TT}$ applies.

^bObtained from the comparison of the anti-proton's frequency with the hydrogen ion's frequency.

^cEstimated value based on the sensitivity calculations of some planned space-experiments.

^dObtained from the neutron.

^eFrom b_μ^{neutron} with the use of a two-species noble-gas maser. From b_μ^{electron} , a weaker but cleaner bound of 1.2×10^{-25} can be obtained.

^fThis bound is from the spatial isotropy test of polarized electrons.